Swizzle Inventor

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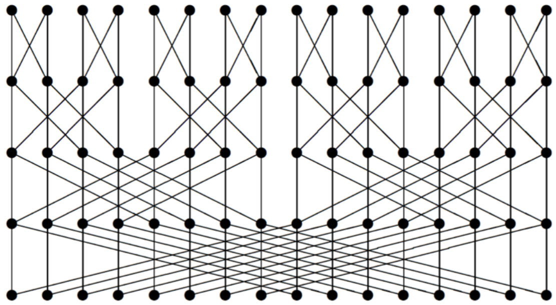
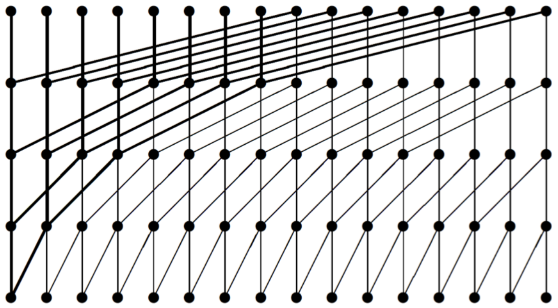
Rastislav Bodik	University of Washington

Swizzle

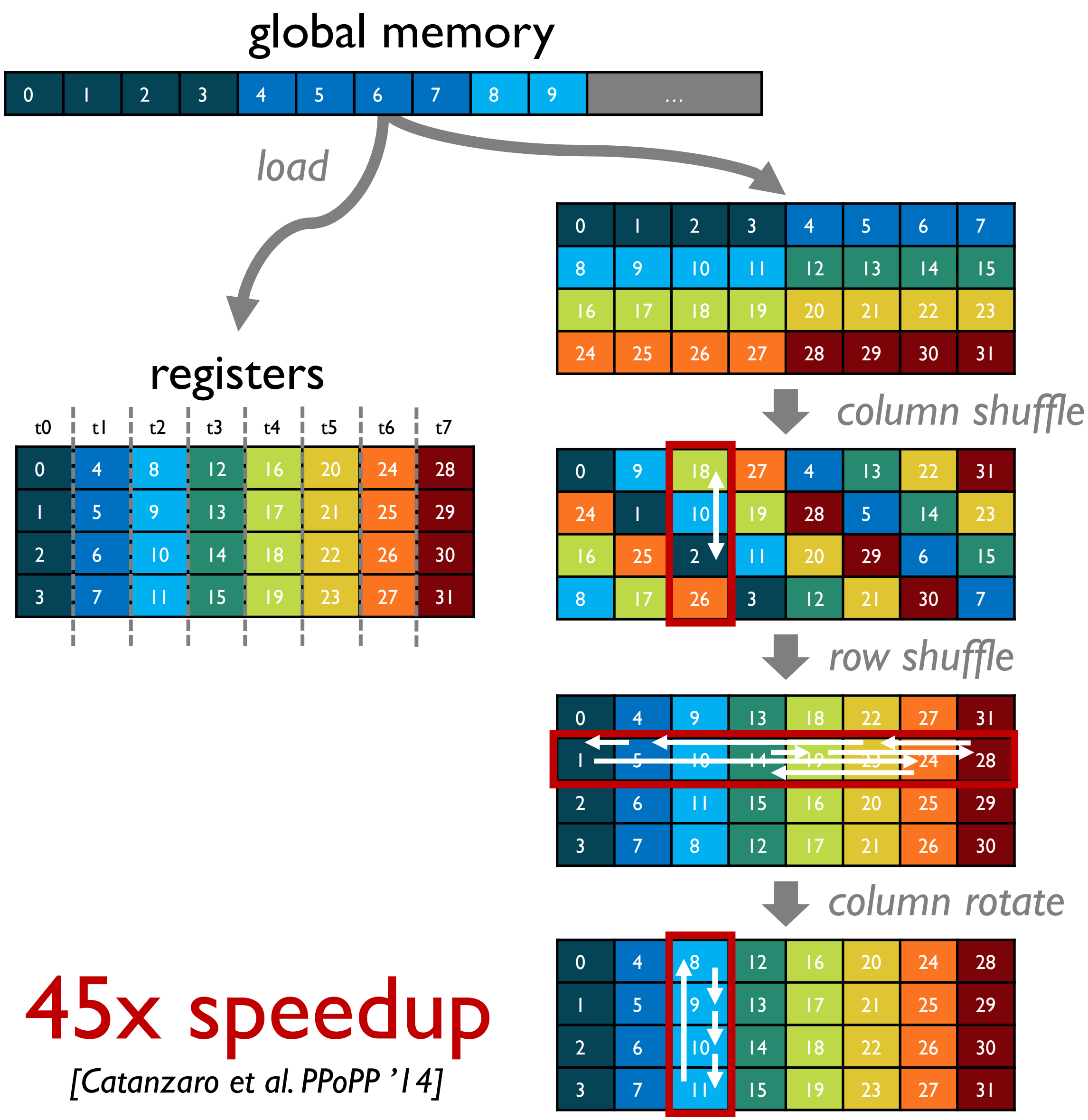
non-trivial movement of data or
non-trivial mapping of computations
to **hardware resources** and **loop iterations**

for dramatic performance improvement



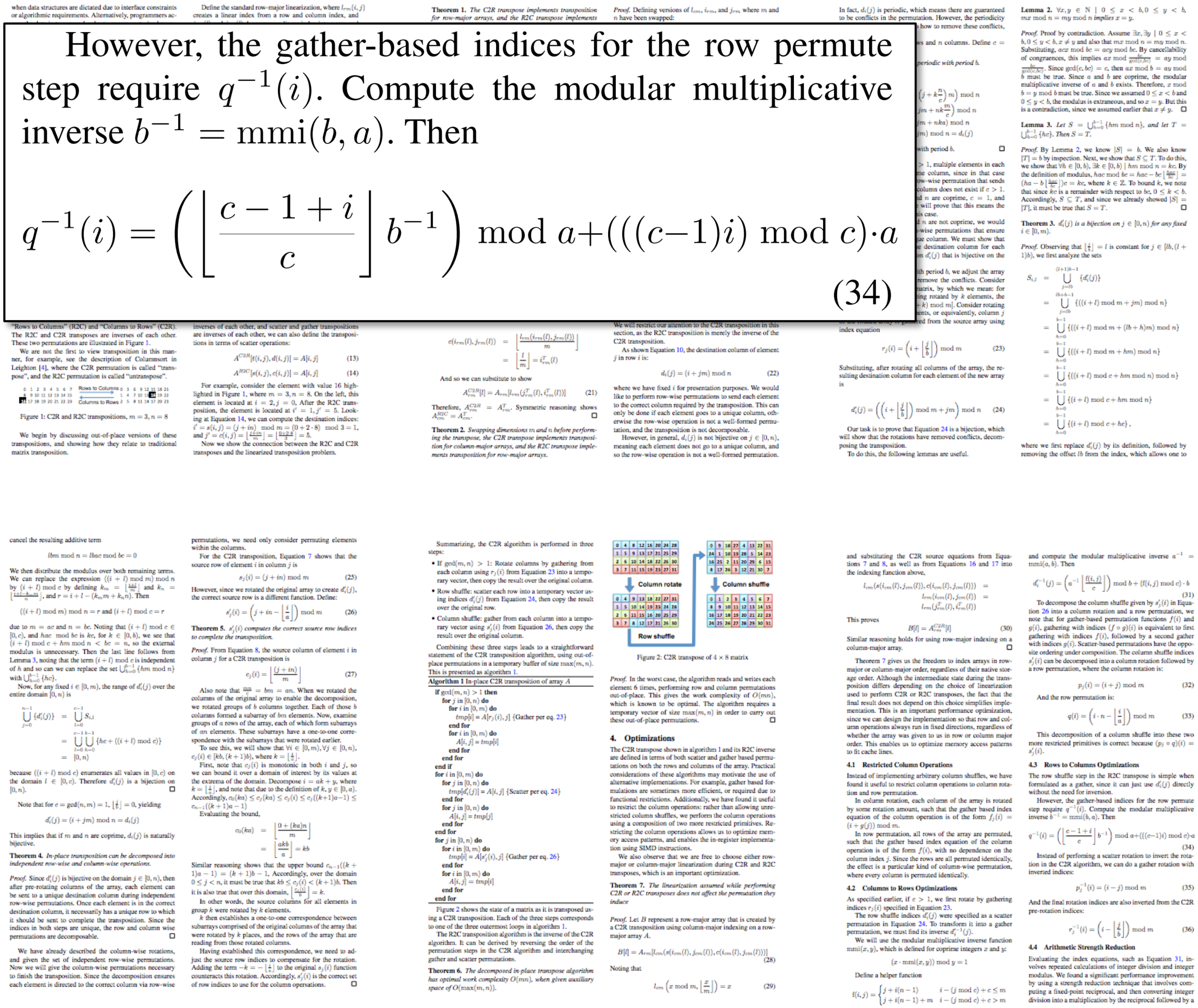


Load Array of Struct in GPU



45x speedup

[Catanzaro et al. PPoPP '14]

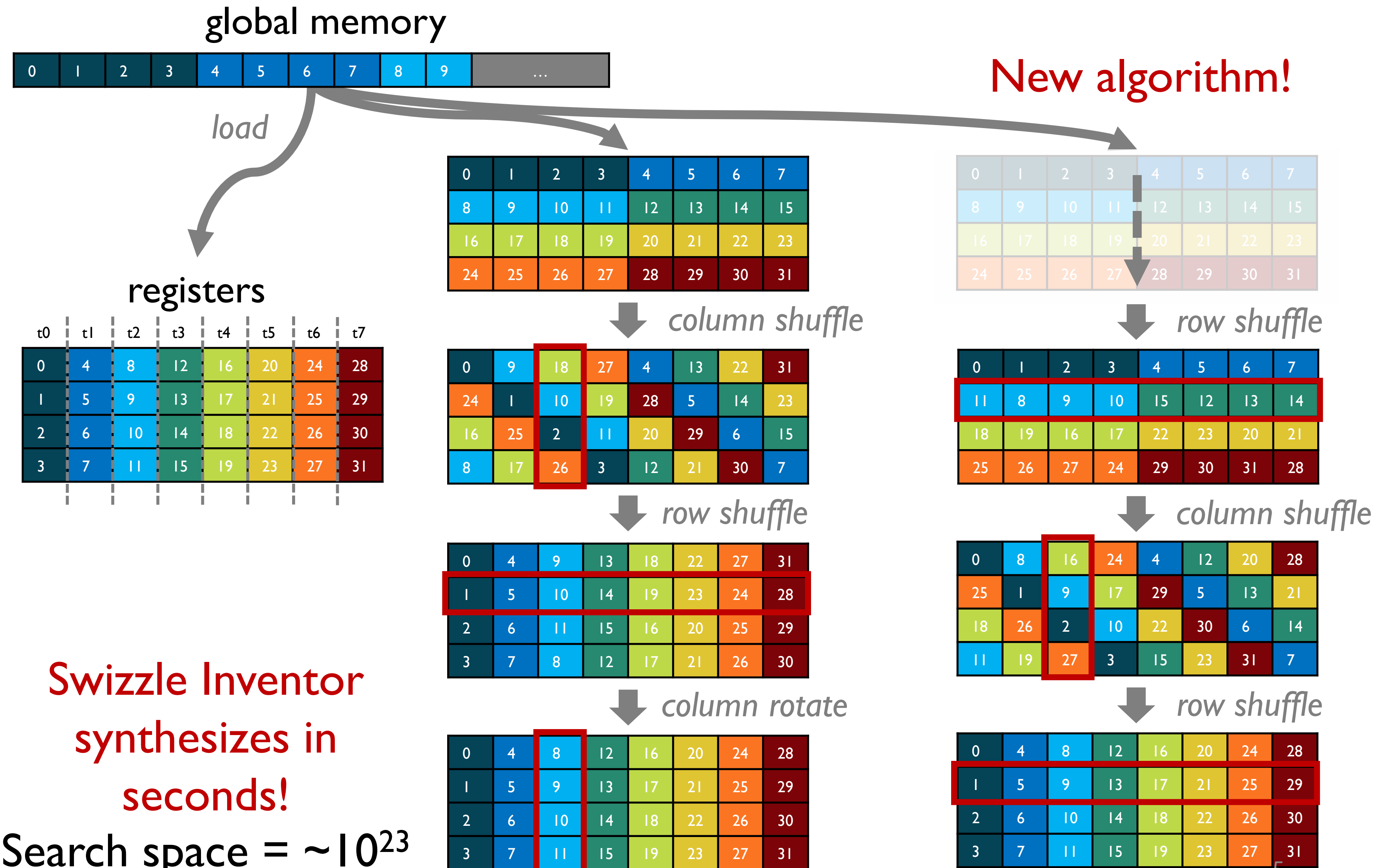


However, the gather-based indices for the row permute step require $q^{-1}(i)$. Compute the modular multiplicative inverse $b^{-1} = \text{mmi}(b, a)$. Then

$$q^{-1}(i) = \left(\left\lfloor \frac{c-1+i}{c} \right\rfloor b^{-1} \right) \bmod a + (((c-1)i) \bmod c) \cdot a \tag{34}$$

[Catanzaro et al. PPoPP '14]

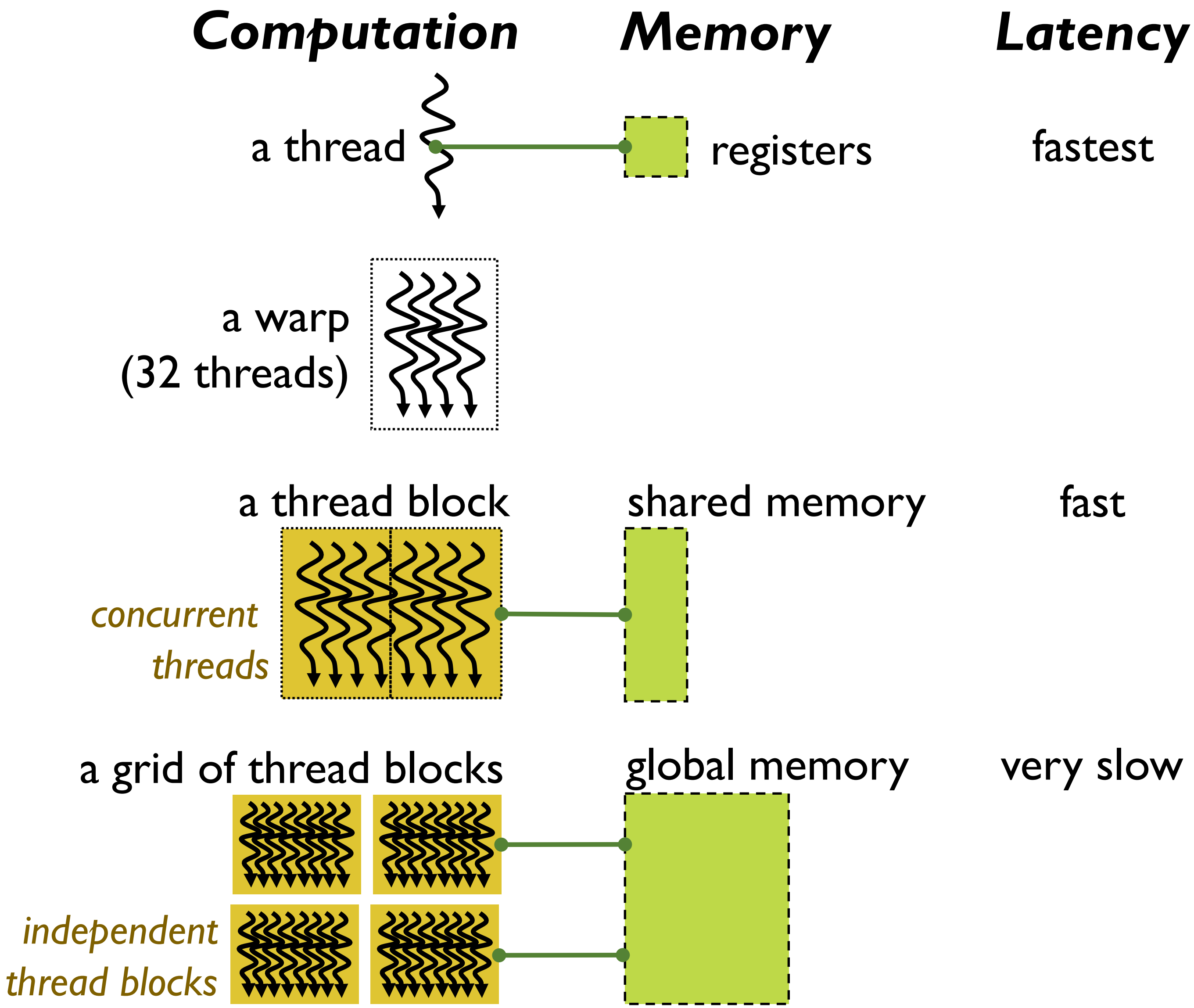
Load Array of Struct in GPU



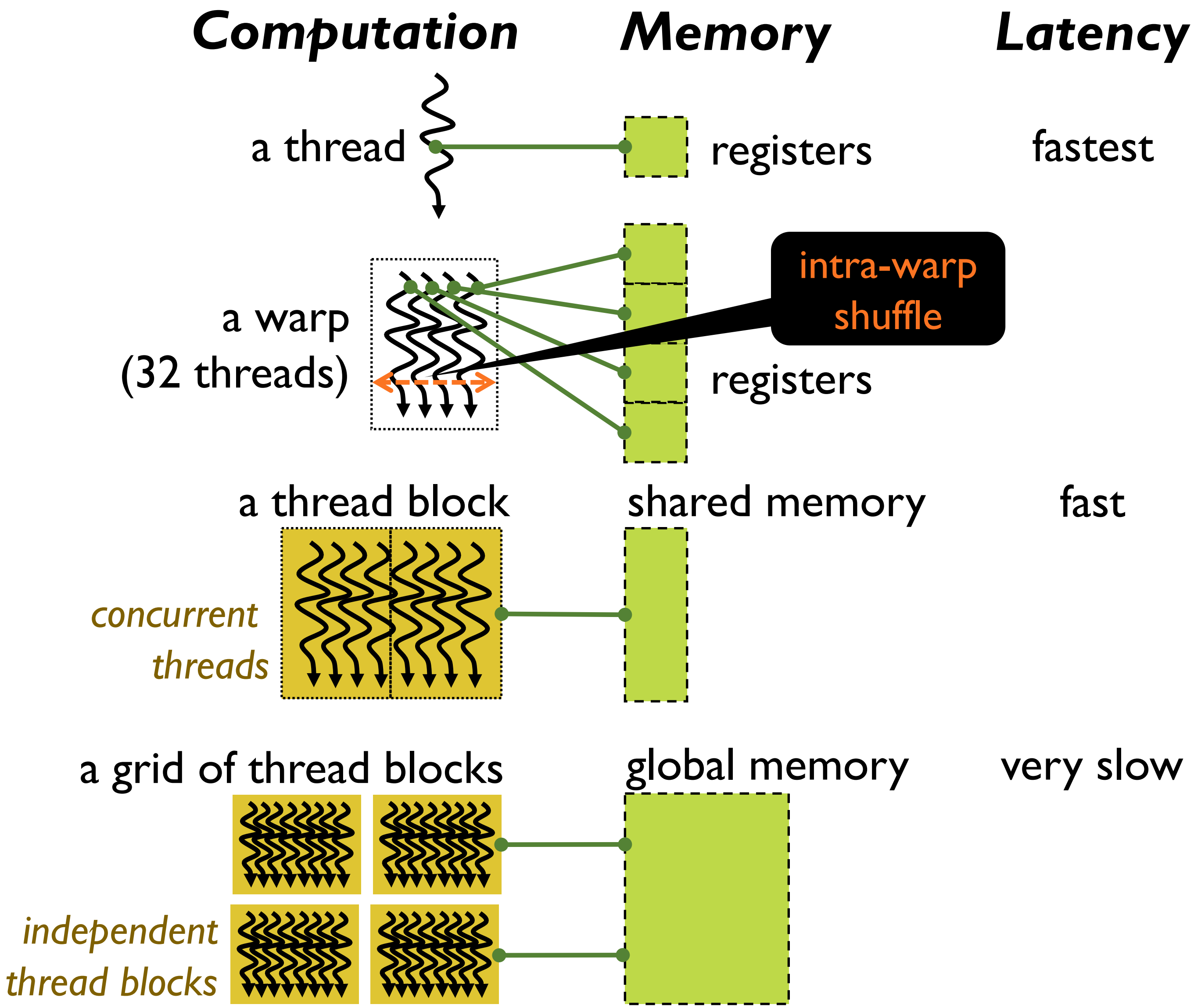
Swizzle Inventor synthesizes in seconds!

Search space = ~10^{23}

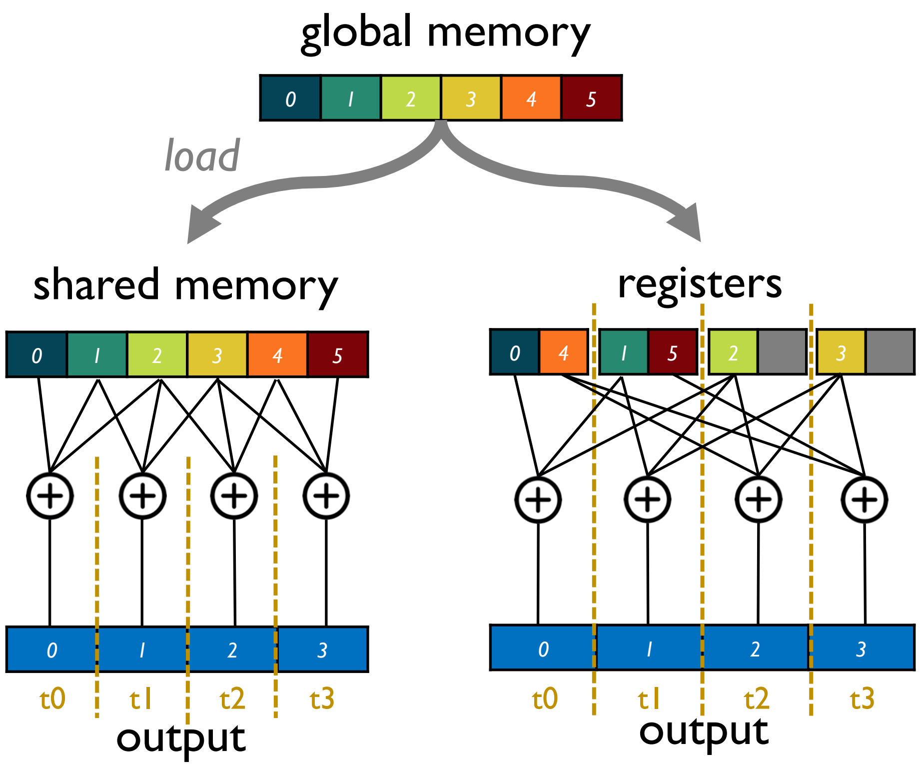
GPU Architecture Basic



GPU Architecture Basic

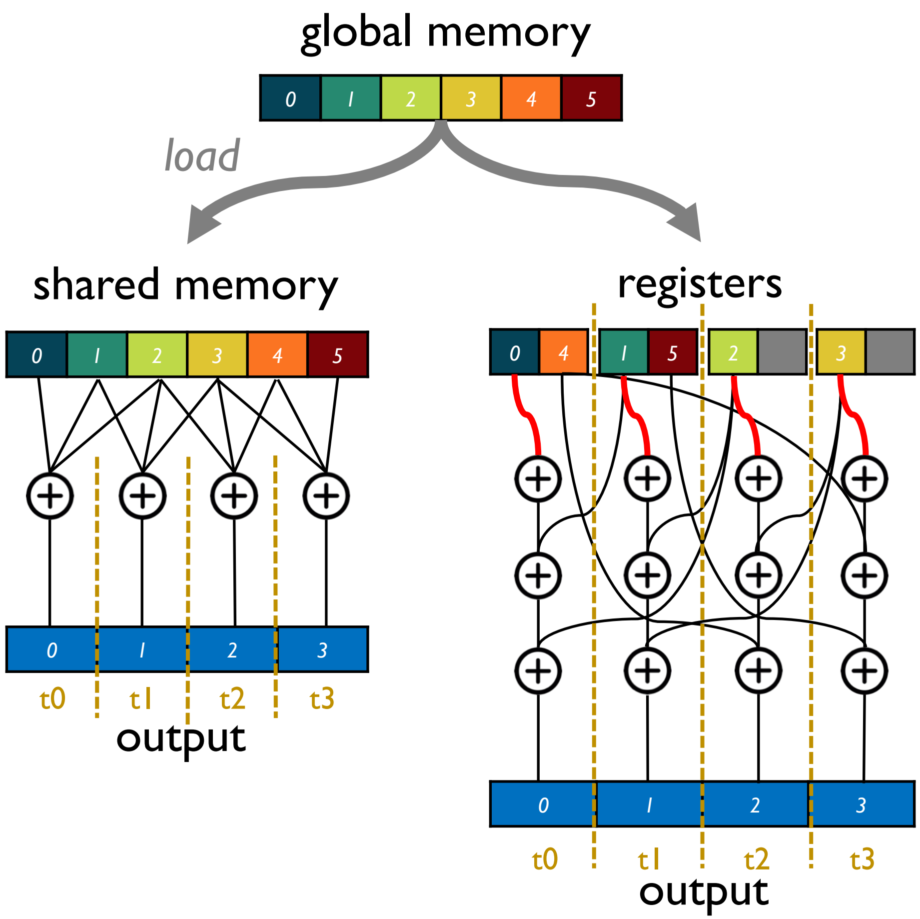


Register Cache: Stencil



[Ben-Sasson et al. ICS' 16]

Register Cache: Stencil



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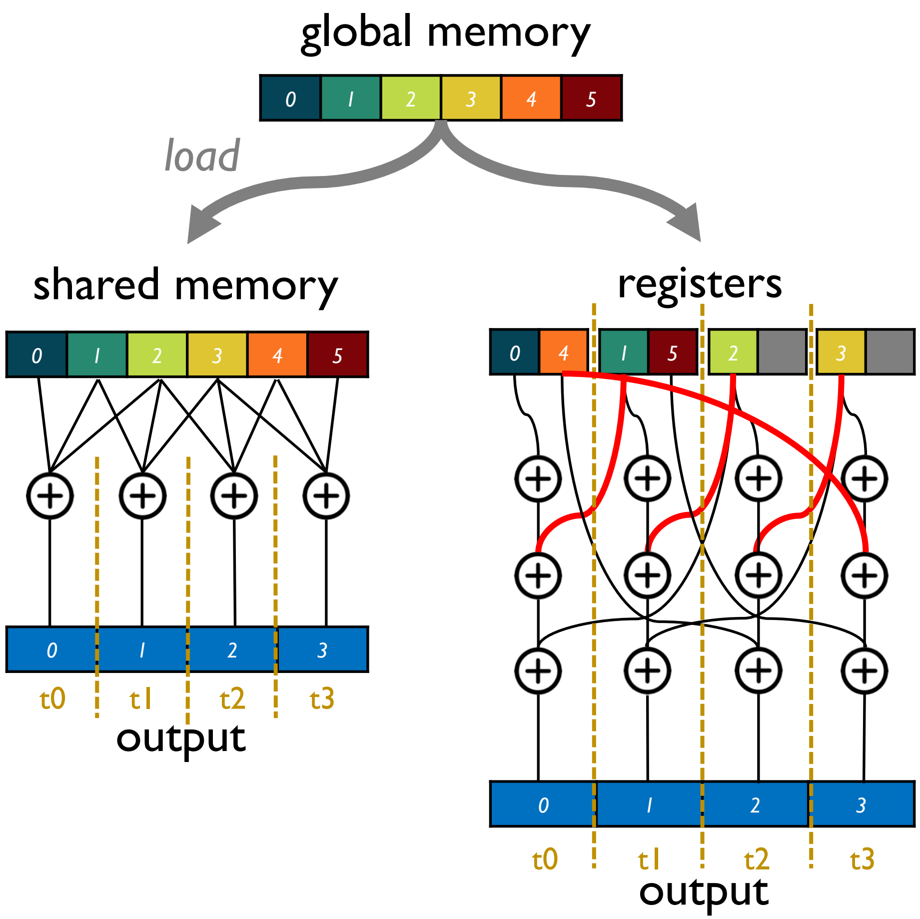

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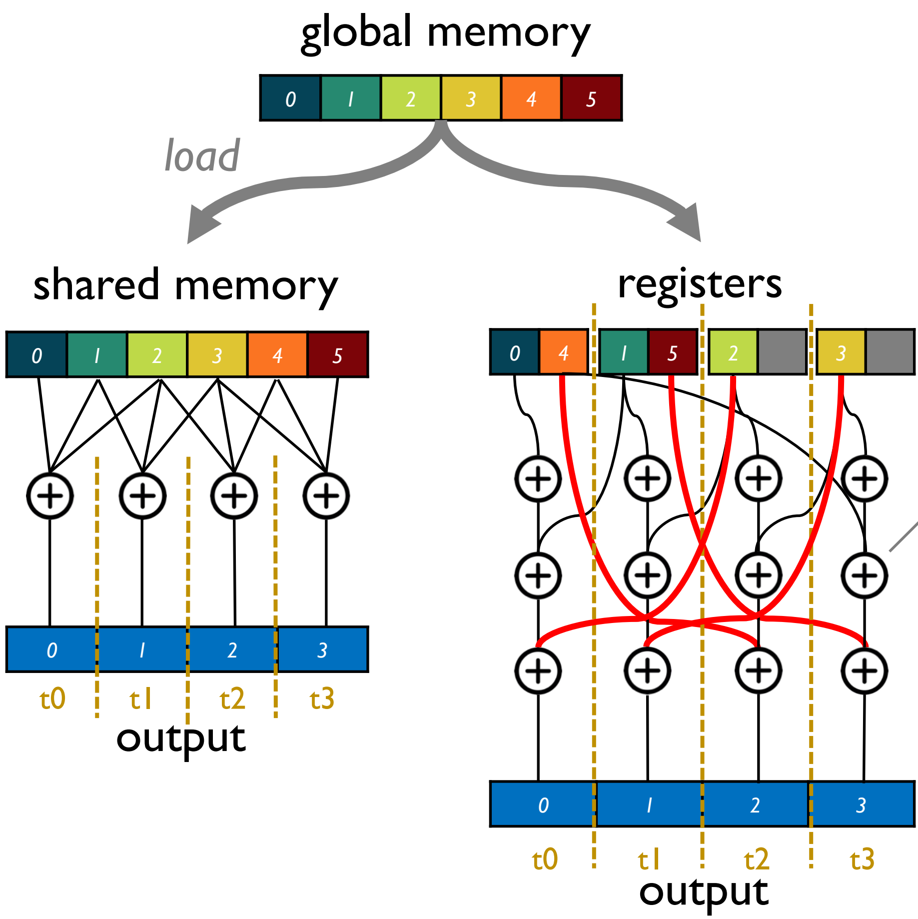
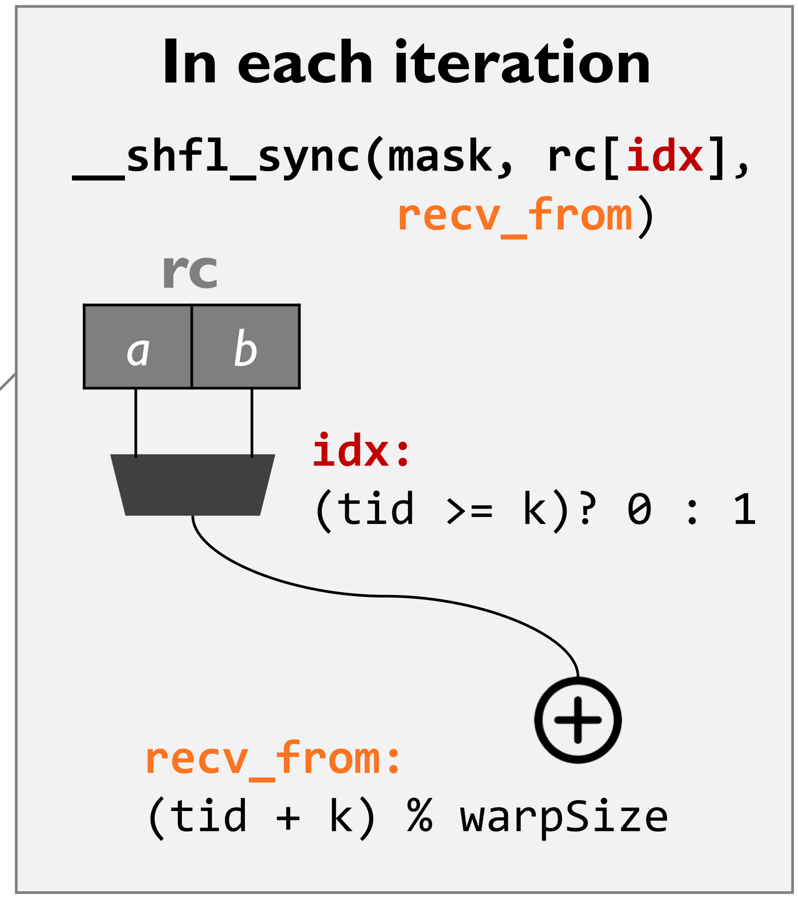




[Ben-Sasson et al. ICS' 16]

Automatic Optimization

These optimizations require:

- reasoning about program globally
- solving multiple constraints together
- rewriting multiple program fragments simultaneously

Cannot be done by a typical rewrite rule in a compiler.

Swizzle Inventor

Helps programmers implement swizzle programs by:

- letting them **write program sketches that omit swizzles**
- **automatically synthesizing swizzles** to complete the programs

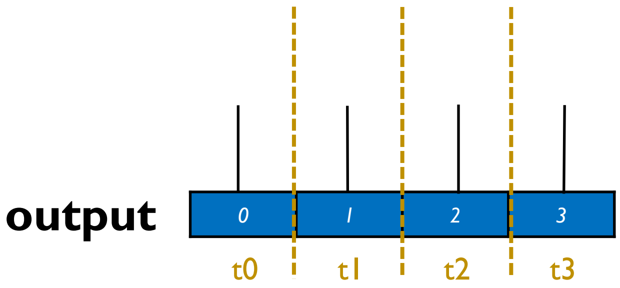
Stencil: Program Sketch

SIMT program

```
rc = load(input, warpOffset,
          /* slice */ 1,
          /* iterations */ 2);

int out = 0;
for(int k = 0; k < 3; k++) {
    int tmp = magic_get(rc);
    out += tmp;
}

output[tid] = out;
```



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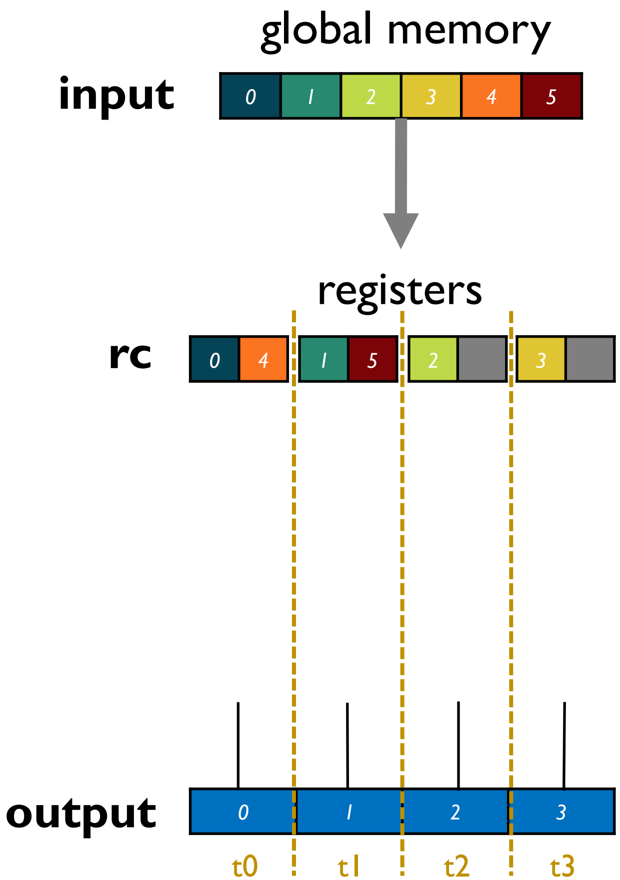

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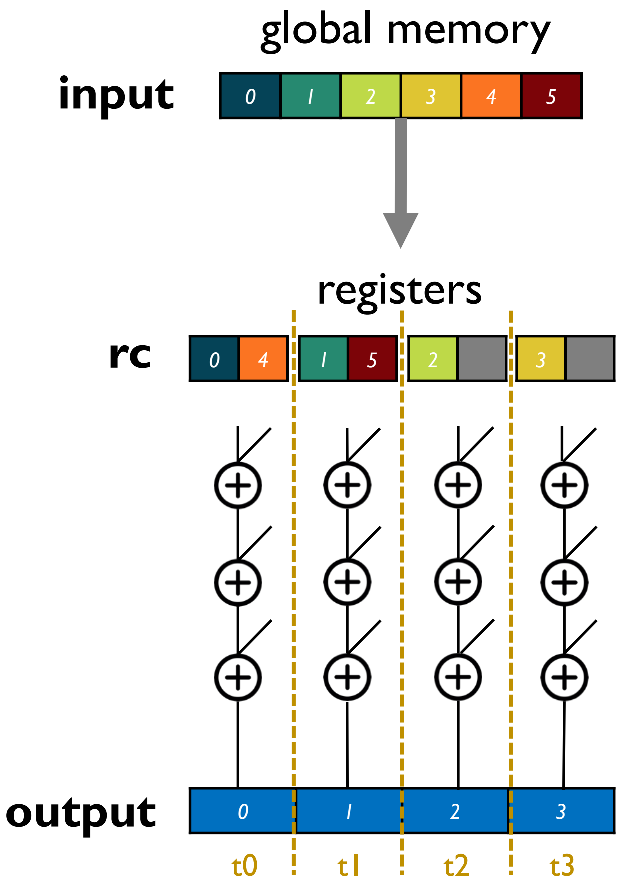

SIMT program

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int out = 0;
for(int k = 0; k < 3; k++) {
    int tmp = magic_get(rc);
    out += tmp;
}

output[tid] = out;
```

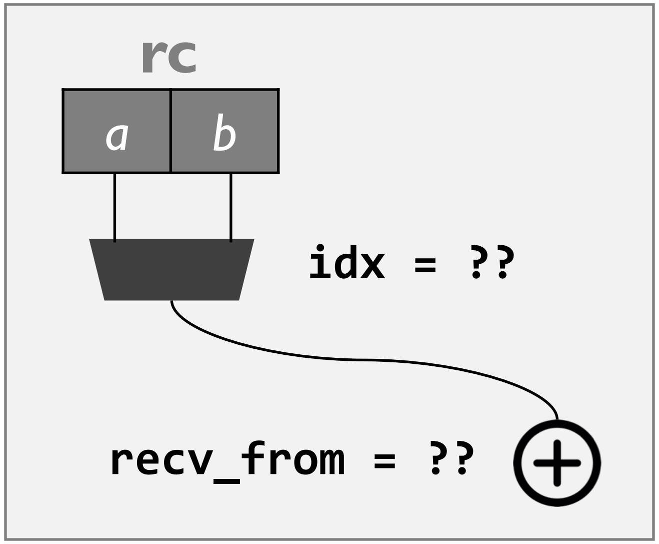
Stencil: Program Sketch

```
int tmp = magic_get(rc); -->

// Choose which input data to send
int idx = ?sw_part(2, tid, k);

// Choose which thread to read from
int recv_from =
?sw_xform(tid, warpSize, k);

// Perform intra-warp shuffle
int tmp = __shfl_sync(FULL_MASK, rc[idx], recv_from);
```



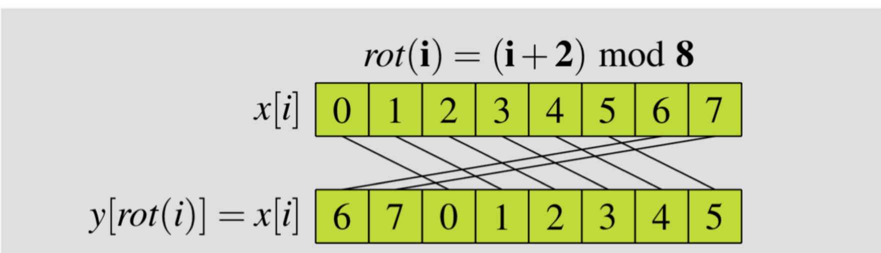
Use `?sw_xform` (transformation swizzle) when `recv_from` is permutation or broadcast of `tid`

Use `?sw_part` (partition swizzle) otherwise

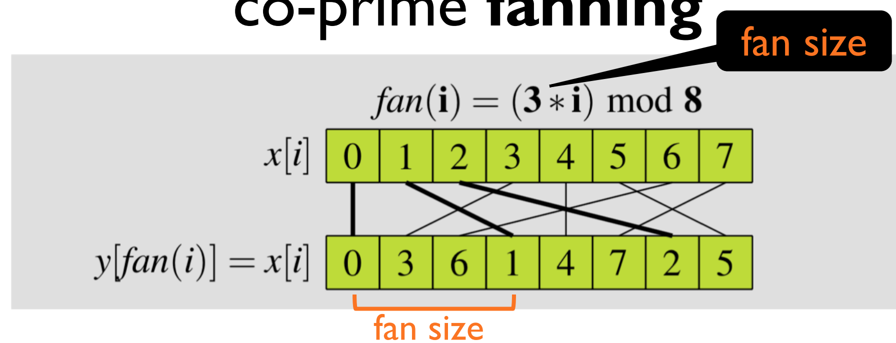
Transformation Swizzle Hole

?sw_xform hole defines the search space that contains **grouping** permutations of **fanning** followed by **rotation**.

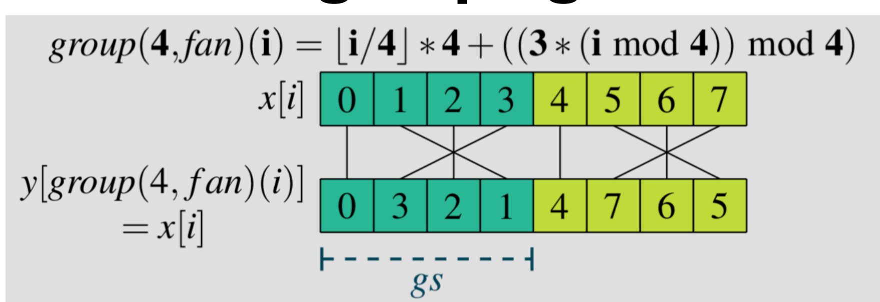
rotation



co-prime **fanning**



grouping



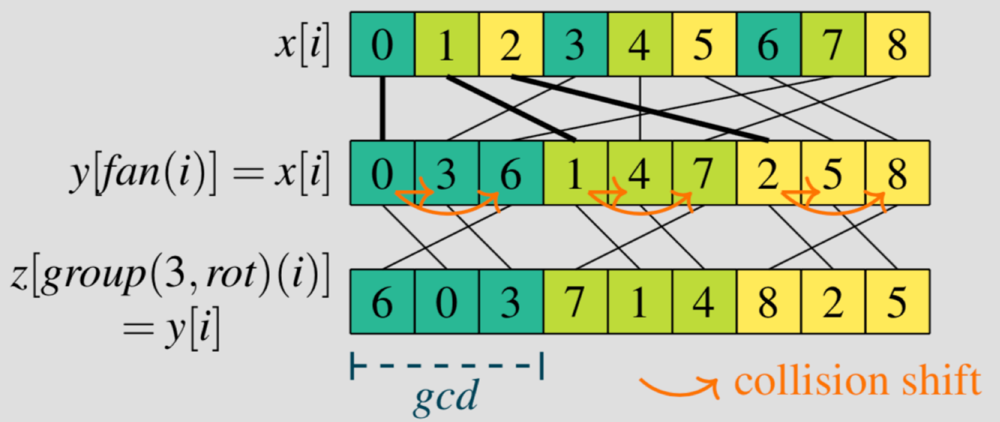
Transformation Swizzle: Example

fanning followed by **grouped rotation**

$$\mathbf{z}[(group(\mathbf{3}, rot) \circ fan)(\mathbf{i})] = \mathbf{x}[\mathbf{i}]$$

$$fan(\mathbf{i}) = (\mathbf{3} * \mathbf{i} + \lfloor \mathbf{i}/\mathbf{3} \rfloor) \bmod \mathbf{9}$$

$$group(\mathbf{3}, rot)(\mathbf{i}) = \lfloor \mathbf{i}/\mathbf{3} \rfloor * \mathbf{3} + (((\mathbf{i} \bmod \mathbf{3}) + \mathbf{1}) \bmod \mathbf{3})$$



Partition Swizzle Hole

```
?sw_part(n, v, …) :=
     if ?sw_cond(v, …)     then 0
     elif ?sw_cond(v, …)   then 1
     ...
     else n - 1
```

Condition Swizzle Hole

$$
\begin{aligned}
\textbf{?sw\_cond}(\mathtt{v}, \ldots) &:= (\mathtt{v} \mid \ldots) \odot_{\mathtt{cmp}} (\mathtt{I} \odot_{\mathtt{bin}} (\mathtt{v} \mid \ldots)) \\
\odot_{\mathtt{cmp}} &:= \; = \mid \neq \mid \geq \mid > \mid \leq \mid < \\
\odot_{\mathtt{bin}} &:= + \mid - \\
\mathtt{I} &:= \mathtt{integer}
\end{aligned}
$$

Correctness Condition

Spec: sequential program

```
void spec(
    const float *x,
    float *y, int n) {

  for(int i = 0; i < n; i++) {
    int out = 0;
    for(int k = 0; k < 3; k++)
      out += x[i+k];
    y[i] = out;
  }
}
```

$$\exists \boldsymbol{h}\, \forall \boldsymbol{x}\,.\, \boldsymbol{spec}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{n}) \\ \wedge\, \boldsymbol{sketch}(\boldsymbol{h})(\boldsymbol{x}, \boldsymbol{y}', \boldsymbol{n}) \\ \wedge\, \mathbf{y} = \mathbf{y}'$$

Sketch: CUDA sketch

```
__global__ void sketch(
    const float *x,
    float *y, int n) {

  rc = load(x, warpOffset, 1, 2);

  int out = 0;
  for(int k = 0; k < 3; k++) {
    int tmp = magic_get(rc);
    out += tmp;
  }

  y[tid] = out;
}
```

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    y[i] = out;
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}
```

$$\exists h \, \forall x \, . \, spec(x, y, n) \\ \wedge \, sketch(h)(x, y', n) \\ \wedge \, y = y'$$

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```

$$\exists h \,.\, spec(\tilde{x}, y, n) \wedge sketch(h)(\tilde{x}, y', n) \wedge \mathbf{y} = \mathbf{y}'$$

array of symbolic variables

Symbolic Representation

x

x_0	x_1	x_2	x_3	x_4	x_5

Spec:

```
for(int i = 0; i < n; i++) {
  int out = 0;
  for(int k = 0; k < 3; k++)
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}
```

$x_0+x_1+x_2$	$x_1+x_2+x_3$	$x_2+x_3+x_4$	$x_3+x_4+x_5$

Sketch:

```
rc = load(x, warpOffset, 1, 2);

int out = 0;
for(int k = 0; k < 3; k++) {
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$x_?+x_?+x_?$	$x_?+x_?+x_?$	$x_?+x_?+x_?$	$x_?+x_?+x_?$

Symbolic Representation

x

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for(int i = 0; i < n; i++) {
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Sketch:

```
rc = load(x, warpOffset, 1, 2);

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  out += tmp;
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```

$x_?+x_?+x_?$	$x_?+x_?+x_?$	$x_?+x_?+x_?$	$x_?+x_?+x_?$

Symbolic Representation

x

x_0	x_1	x_2	x_3	x_4	x_5

Spec:

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for(int i = 0; i < n; i++) {
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  for(int k = 0; k < 3; k++)
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  y[i] = out;
}
```

$\{x_0, x_1, x_2\}$ op: +	$\{x_1, x_2, x_3\}$ op: +	$\{x_2, x_3, x_4\}$ op: +	$\{x_3, x_4, x_5\}$ op: +

Sketch:

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$\{x_?, x_?, x_?\}$ op: +	$\{x_?, x_?, x_?\}$ op: +	$\{x_?, x_?, x_?\}$ op: +	$\{x_?, x_?, x_?\}$ op: +

Symbolic Representation

x

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$\{x_?, x_?, x_?\}$ op: +	$\{x_?, x_?, x_?\}$ op: +	$\{x_?, x_?, x_?\}$ op: +	$\{x_?, x_?, x_?\}$ op: +

≡ *multiset*

Accumulator

x

x_0	x_1	x_2	x_3	x_4	x_5

Spec:

```
for(int i = 0; i < n; i++) {
  o = create_accumulator(0, +);
  for(int k = 0; k < 3; k++)
    accumulate(o, [x[i+k]], true);
  y[i] = eval(o);
}
```

$\{x_0,x_1,x_2\}$ op: +	$\{x_1,x_2,x_3\}$ op: +	$\{x_2,x_3,x_4\}$ op: +	$\{x_3,x_4,x_5\}$ op: +

Sketch:

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```

$\{x_?,x_?,x_?\}$ op: +	$\{x_?,x_?,x_?\}$ op: +	$\{x_?,x_?,x_?\}$ op: +	$\{x_?,x_?,x_?\}$ op: +

$\equiv$ ***multiset***

Accumulator

Sum stencil: $\bigoplus \rightarrow +$

$$\{x, y, x\} \equiv_{multiset} \{x, x, y\}$$

$$x + y + x = x + x + y$$

Convolution: $\bigoplus \rightarrow +$, $\bigodot \rightarrow \times$

$$\{\{w, x\}, \{u, y\}\} \equiv_{multiset} \{\{u, y\}, \{x, w\}\}$$

$$(w \times x) + (u \times y) = (u \times y) + (x \times w)$$

$\bigoplus$ and $\bigodot$ must be
associative and **communitive**.

Search Problem

Spec: sequential program

```
void spec(
    const float *x,
    float *y, int n) {

  for(int i = 0; i < n; i++) {
    o = create_accumulator(0,identiy,+);
    for(int k = 0; k < 3; k++)
      accumulate(o, [x[i+k]], true);
    y[i] = eval(o);
  }
}
```

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    int tmp = magic_get(rc);
    accumulate(o, [tmp], true);
  }

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}
```

Expressiveness:

Can Swizzle Inventor synthesize GPU kernels with swizzling optimizations in the literature?

Stencil computations

Finite field multiplication

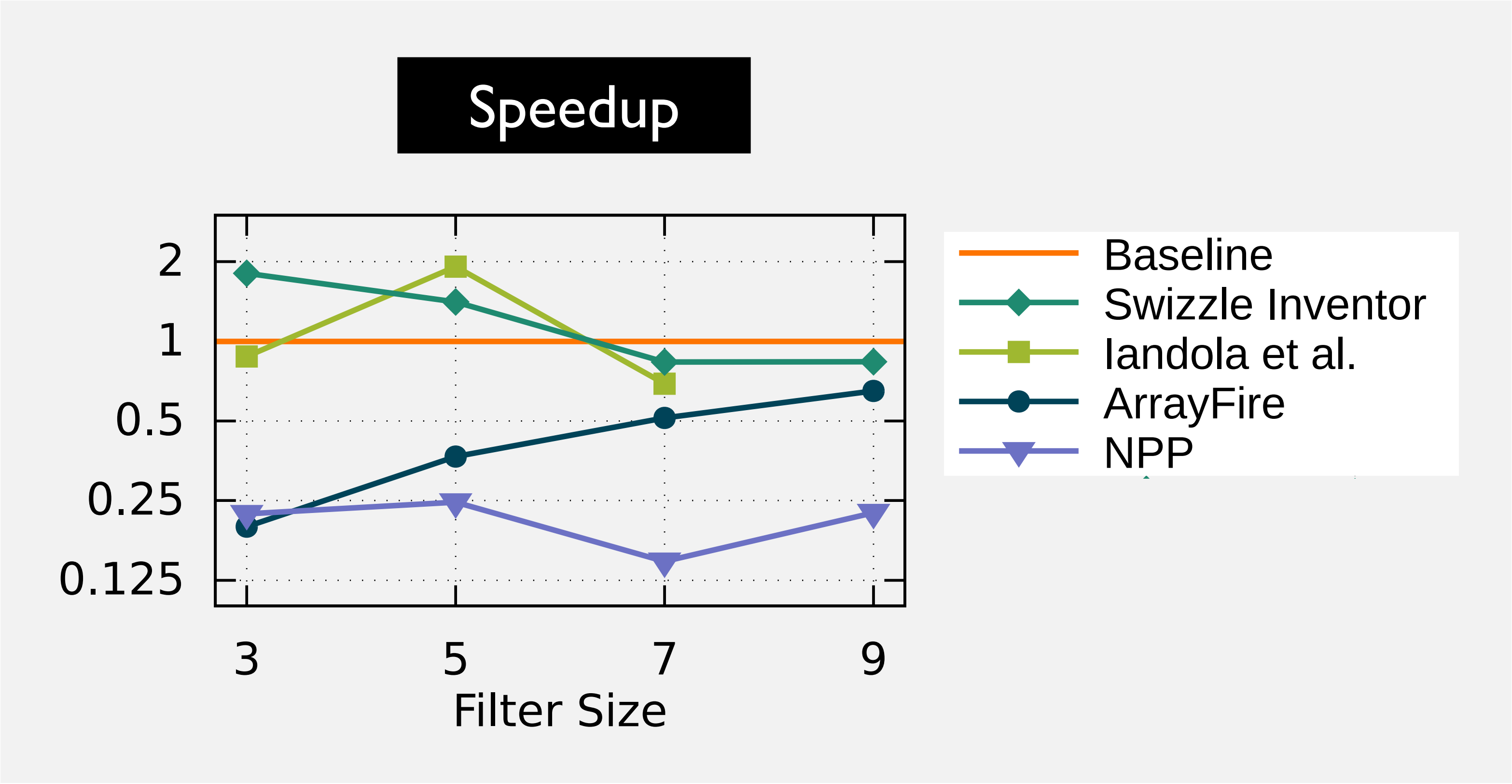
Matrix transposition

Inventiveness:

Can Swizzle Inventor invent new optimizations?

Stencil: 2D Convolution

Use registers to cache input image.



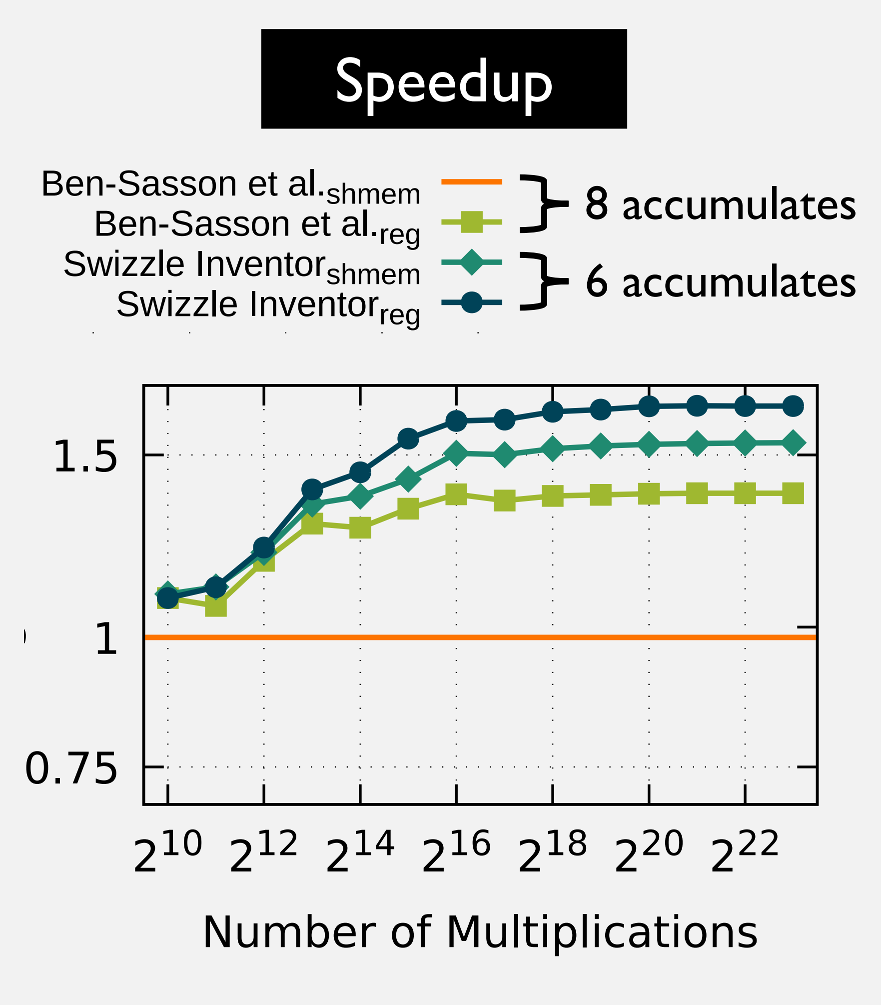
Finite Field Multiplication

```
// Create ans0, ans1, ans2, ans3
acc ans0 = create_accumulator(0, identity, ^, &);
...

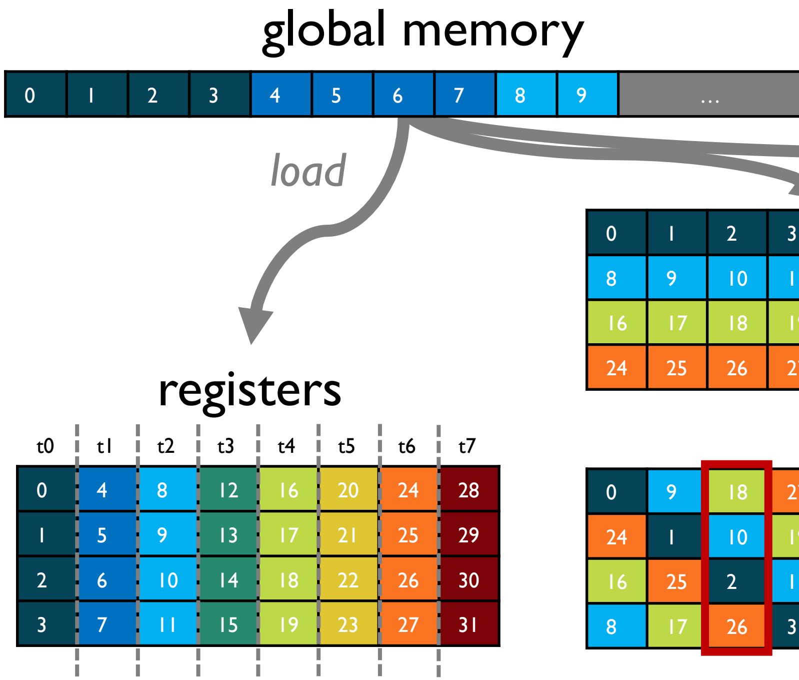
for(int k = 0; k < 32; k++) {
  int a0 = __shfl_sync(mask, rA[?sw_part(2,tid,k)],
                       ?sw_xform(tid,32,k));
  int a1 = __shfl_sync(mask, rA[?sw_part(2,tid,k)],
                       ?sw_xform(tid,32,k));
  int b0 = __shfl_sync(mask, rB[?sw_part(2,tid,k)],
                       ?sw_xform(tid,32,k));
  int b1 = __shfl_sync(mask, rB[?sw_part(2,tid,k)],
                       ?sw_xform(tid,32,k));

  // Update ans0
  accumulate(ans0, [a0,b0], ?sw_cond(tid,k));
  accumulate(ans0, [a0,b1], ?sw_cond(tid,k));
  accumulate(ans0, [a1,b0], ?sw_cond(tid,k));
  accumulate(ans0, [a1,b1], ?sw_cond(tid,k));

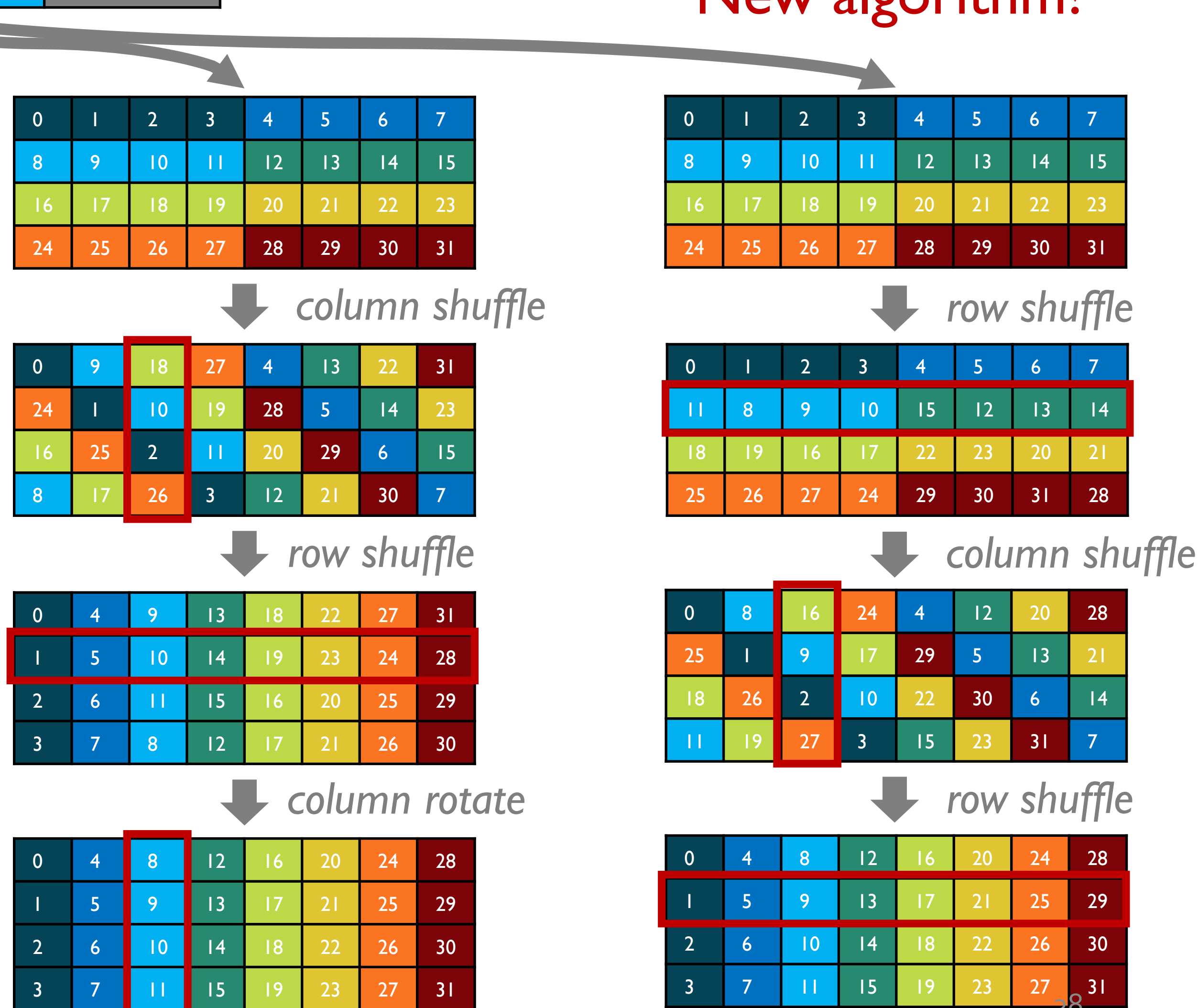
  // Update ans1, ans2, ans3
  ...
}
```



Matrix Transposition



New algorithm!



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github.com/mangpo/swizzle-inventor